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COMPARISON OF METHODS FOR RANDOM VIBRATIONS CALCULATIONS OF MECHANICAL SYSTEMS

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Abstract. *This paper presents comparison of the methods for random vibrations calculations implemented in programs Ansys and Imita. It is shown that for the examined cases of narrow band random excitation the accuracy of Ansys essentially depends on dissipation of the dynamic system while results obtained by Imita practically coincide with the exact analytical solutions. The main aim is the investigation of the usability of these methods in the automated procedures for random vibrations response optimisation.*

1 Introduction

Nowadays virtual prototyping has become a very essential part of the designing process. A large number of various CAD/CAE programs are available. In the Internet now one can find at least one hundred commercial software tools based on finite element method (FEM) that are useful for solution of a broad class of the different engineer tasks. In the most cases these programs have compatible data interfaces with one another as well as with popular CAD software. A considerably smaller number of programs are available for the analysis of non-linear mechanical system dynamics. One of the most powerful is the Adams program that now is intensively developed and simultaneously integrated with the railway vehicle dynamics analysis program Medyna. A comparison of Medyna with other software tools including programs LMS and Imita developed in Riga Technical University you can find in the book [1]. We must note that the above given division is not strict. With FEM software tools we can solve some dynamic problems while dynamic analysis software is supplemented with the FEM modules. Additionally these tools are often equipped with optimisation modules.

An alternative tool Imita [2] is considered for vibration response optimisation of nonlinear mechanical systems. The investigations are carried out for the objects, dynamic schemes of which it is possible to construct from rigid bodies that are interlinked by means of non-inertial elastic and damping links. Small spatial oscillations are considered. Recently, it was shown [3, 4] that the optimisation tasks of the dynamical systems without pronounced criteria and with great number of constraints are possible to solve effectively by using global optimisation systems and specialised three-step strategy for localisation of the solution. Accordingly, those regions are localised in the first step, where the pure constraints are satisfied. In the second step, further localisation of the regions is obtained where the side constraints of the minimised state variables are satisfied. In the last step, the minimised state variables are united by means of binding intervals. Such binding in dialog mode gives possibility for the users by changing of the appropriate values of the binding interval parameters to easily manage searching for the compromise solution in the necessary direction.

Nevertheless, it is crucial to diminish expenses for each trial point calculation, especially for time-consuming cases of random vibration investigations. For this we try to investigate the possibilities to use time domain methods for random vibrations calculations in an optimisation loop. Method of the correlation differential equations solution [5] is examined in detail. Accuracy of the results obtained by the proposed time domain method and the power spectral density method is compared with the exact analytical solution.

2 Solution of Equations of Motion

One has solved various types of differential equations to obtain dynamic response of the examined class of discrete mechanical systems. Let us consider one of the methods used in Imita for investigation of random vibrations. Suppose that dynamic response of the discrete mechanical systems is described by linearised equation:

$$A\ddot{q} + B\dot{q} + Cq = f(t), \tag{1}$$

where A, B and C are inertial, damping and stiffness matrices of order n,
f, q are column vectors of stationary and stationary bonded excitations and generalised coordinates, respectively.
For determining solution in the time domain one has to solve two differential correlation equations:

$$\begin{aligned} A\ddot{K}_{qf}(\tau) + B\dot{K}_{qf}(\tau) + CK_{qf}(\tau) &= K_{ff}(\tau) \\ A\ddot{K}^{T}_{qq}(\sigma) + B\dot{K}^{T}_{qq}(\sigma) + CK^{T}_{qq}(\sigma) &= K^{T}_{qf}(\sigma) \end{aligned} \tag{2}$$

where
$K_{ff}(\tau)$ - square matrix of correlation functions of excitations f,
$K_{qq}(\tau)$ - square matrix of correlation functions of generalised coordinates q,
$K_{qf}(\tau)$ - square matrix of mutual correlation functions of generalised coordinates q and excitations f,
τ - difference of time instants for which the correlation bonds are evaluated ($\sigma = -\tau$).
Frequently the correlation functions of stationary and stationary bonded random excitations could be approximated by means of expressions of the following type:

$$K_f(\tau) = De^{-\alpha|\tau|}(\cos\beta\tau + \alpha/\beta\sin\beta|\tau|), \tag{3}$$

where parameters D>0, α >0, $\beta \geq 0$. The presence of expressions like (3) in the right - hand part of the first correlation equation (FCE) requires its solution to be obtained in two intervals: I1- for $-\infty \leq \tau \leq 0$ and I2- for $0 \leq \tau \leq \infty$ with subsequent stitching for argument value $\tau = 0$.
The following boundary conditions derive from the properties of the correlation functions

$$K_{qf}(-\infty) = K_{qf}(\infty) = 0. \tag{4}$$

The method for solution of (2) is given, for example, in the article [4] for the case of excitations like (3). It is somewhat cumbersome in comparison with the methods of the spectral density theory.
To inspect algorithm of proposed method carefully, let us consider simple oscillation system described by equation:

$$\ddot{y} + a_1\dot{y} + a_0 y = p, \tag{5}$$

where p is a stationary random process.

2.1 Excitation No. 1

Firstly random excitation described by the appropriate correlation function is considered:

$$K_{pp}(\tau) = De^{-\alpha|\tau|} \tag{6}$$

Then FCE will be

$$\ddot{K}_{yp}(\tau) + a_1\dot{K}_{yp}(\tau) + a_0 K_{yp}(\tau) = De^{-\alpha|\tau|} \tag{7}$$

Solution of the FCE (7) in interval I1 is following

$${}^{-}K_{yp}(\tau) = \frac{D}{\alpha^2 + a_1\alpha + a_0}e^{\alpha\tau} \tag{8}$$

and in I2 subsequent

$${}^{+}K_{yp}(\tau) = C_1 e^{-\frac{a_1}{2}|\tau|}\cos\beta\tau + C_2 e^{-\frac{a_1}{2}|\tau|}\sin\beta\tau + D_{yp}e^{-\alpha\tau} \tag{9}$$

where

$$\beta = \sqrt{a_0 - a_1^2/4} \quad \text{and} \quad D_{yp} = \frac{D}{\alpha^2 - a_1\alpha + a_0} \tag{10}$$

The integration constants C_1 and C_2 can be obtained from stitching condition of FCE in I1 and I2 for argument value τ=0:

$${}^{-}K_{yp}(0) = {}^{+}K_{yp}(0) \quad \text{or} \quad \frac{D}{\alpha^2 + a_1\alpha + a_0} = C_1 + \frac{D}{\alpha^2 - a_1\alpha + a_0} \tag{11}$$

$${}^{-}\dot{K}_{yp}(0) = {}^{+}\dot{K}_{yp}(0) \quad \text{or} \quad \frac{\alpha D}{\alpha^2 + a_1\alpha + a_0} = -\frac{a_1}{2}C_1 + \beta C_2 - \alpha D_{yp} \tag{12}$$

Then we can obtain from the (11)

$$C_1 = \frac{2\alpha a_1 D}{(a_0 + \alpha^2)^2 - a_1^2\alpha^2} \tag{13}$$

and from (12):

$$C_2 = \frac{2\alpha D}{\beta}\frac{\alpha^2 + a_0 - 0{,}5a_1^2}{(a_0+\alpha^2)^2 - a_1^2\alpha^2} \tag{14}$$

Now we have to solve the second correlation equation (SCE) of kind:

$$\ddot{K}_{yy}(\tau) - a_1\dot{K}_{yy}(\tau) + a_0 K_{yy}(\tau) = K_{yp}(\tau) \tag{15}$$

We must seek solution of (15) for argument values $\tau \geq 0$. It will be subsequent:

$$K_{yy}(\tau) = K_{yy1}(\tau) + K_{yy2}(\tau) \tag{16}$$

where $K_{yy1}(\tau)$ and $K_{yy2}(\tau)$ are expressions comprising the appropriate particular solutions for two excitations, namely: 1) complementary function of FCE in I2 and 2) particular solution of FCE in I2.
Firstly we shall obtain $K_{yy1}(\tau)$ by solving of following equation:

$$\ddot{K}_{yy1}(\tau) - a_1\dot{K}_{yy1}(\tau) + a_0 K_{yy1}(\tau) = (C_1\cos\beta\tau + C_2\sin\beta\tau)e^{-\frac{a_1}{2}\tau} \tag{17}$$

Particular solution of (17) we are seeking in the form:

$$K_{yy1}(\tau) = (B_1\cos\beta\tau + B_2\sin\beta\tau)e^{-\frac{a_1}{2}\tau} \tag{18}$$

By substitution of the (18) in the (17) we obtain two linear algebraic equations:

$$\begin{aligned} B_1(a_0 - \beta^2 + \frac{3}{4}a_1^2) + B_2 2a_1\beta &= C_1 \\ B_1 2a_1\beta + B_2(a_0 - \beta^2 + \frac{3}{4}a_1^2) &= C_2 \end{aligned} \tag{19}$$

or taking into account the first relation of (10) we get:

$$\begin{bmatrix} a_1^2 & -2a_1\beta \\ 2a_1\beta & a_1^2 \end{bmatrix}\begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} \text{ and } B_1 = \frac{\alpha D(\alpha^2 + a_0 - a_1^2)}{a_0 a_1[(a_0+\alpha^2)^2 - \alpha^2 a_1^2]}. \tag{20}$$

For the second part of excitation we have to consider equation

$$\ddot{K}_{yy2}(\tau) - a_1 \dot{K}_{yy2}(\tau) + a_0 K_{yy2}(\tau) = D_{yp} e^{-\alpha\tau}, \tag{21}$$

particular solution of which we are seeking in the form:

$$K_{yy2}(\tau) = D_{yy} e^{-\alpha\tau} \tag{22}$$

By substitution of the (22) in the (21) we obtain:

$$\alpha^2 D_{yy} + a_1 \alpha D_{yy} + a_0 D_{yy} = \frac{D}{\alpha^2 - \alpha a_1 + a_0} \quad \text{and}$$

$$D_{yy} = \frac{D}{(\alpha^2 + a_0 + \alpha a_1)(\alpha^2 + a_0 - \alpha a_1)} \tag{23}$$

So particular solution of (15) is following:

$$K_{yy}(\tau) = \frac{D}{(\alpha^2 + a_0)^2 - \alpha a_1^2]} e^{-\alpha\tau} + \left[\frac{\alpha D(\alpha^2 + a_0 - a_1^2)}{a_0 a_1 [(a_0 + \alpha^2)^2 - \alpha^2 a_1^2]} \cos\beta\tau + B_2 \sin\beta\tau \right] e^{-\frac{a_1}{2}\tau}$$

Now dispersion of output process y we obtain by substitution of argument $\tau = 0$:

$$D_y = \frac{D[a_0 a_1 + \alpha(\alpha^2 + a_0 - a_1^2)]}{a_0 a_1 [(a_0 + \alpha^2)^2 - \alpha^2 a_1)}$$

or after simplification:

$$D_y = \frac{D(\alpha + a_1)}{a_0 a_1 (a_0 + \alpha^2 + \alpha a_1)} \tag{24}$$

The relation (24) fully corresponds with result obtained by method of spectral density theory, see, for example, relation (92.19) in [6].

Rough block scheme of algorithm is shown on the fig. 1. As we can see, it is necessary to obtain the complementary function of FCE for I2 as well as the particular solutions of FCE for I1 and I2. If we analyse mechanical system with n D-o-F and one excitation, then the first one can be obtained by solution of the standard eigenvalue problem for an arbitrary real matrix of order 2n and by solution of the linear algebraic equations of order 2n (for calculation of the integration constants). The second one can be obtained by solution of linear

algebraic equations of order 2n for two times, namely, for I1 and I2. To get dispersion of the output process, it is necessary to obtain particular solutions of SCE for (n+1) times, that is get solution of (n+1) equations like (19).

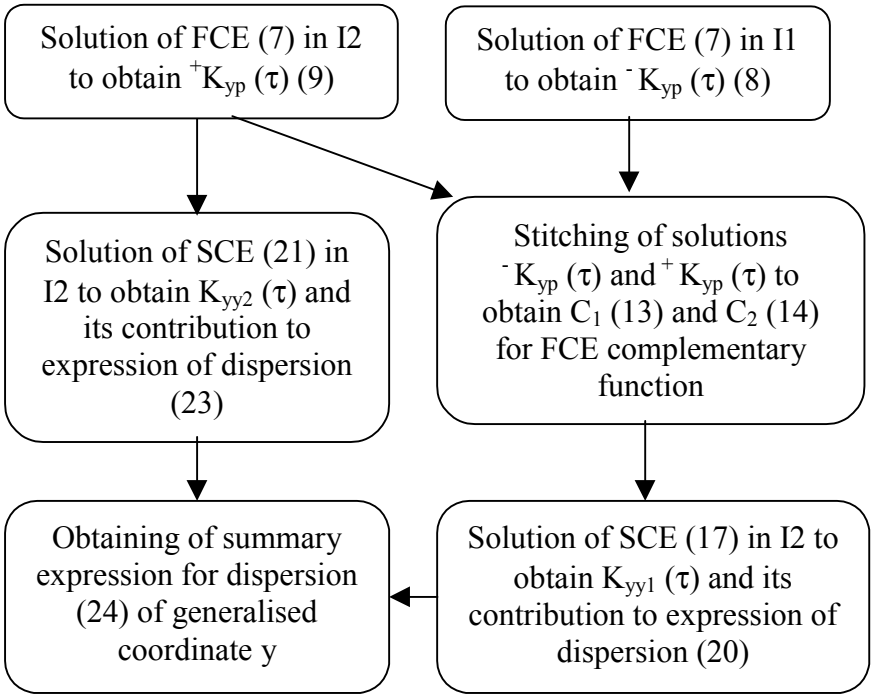

Figure 1: Simplified block scheme of algorithm

So proposed algorithm needs solution of (n+4) algebraic equations of order 2n. From the computational point of view it is more effective to use subsequent inverse matrices for solution of the equations, because 1) for the case of quadratic matrix of excitation $K_{ff}(\tau)$ in the right - hand part of FCE (2) the subsequent inverse matrices can be used repeatedly for every excitation column and 2) it is possible to use inverse matrix, obtained for solution of FCE in I1, for solution of SCE in I2. The structure of these matrices is similar. Thus for solution of SCE we have to inverse the corresponding matrices only n times instead of (n+1) times.

2.2 Excitation No. 2

As a next let us consider excitation described by subsequent correlation function:

$$K_{pp}(\tau) = De^{-\alpha|\tau|}\cos\beta_0\tau\,. \tag{25}$$

The spectral density theory method let us use now. Power spectral density function of excitation we can obtain from the relation:

$$S_p(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega\tau} K_{pp}(\tau)d\tau . \quad (26)$$

By substitution of the (25) in the (26) and integration we obtain

$$S_p(\omega) = \frac{D\alpha}{\pi}\frac{\omega^2+\gamma}{|(i\omega)^2+2\alpha(i\omega)+\gamma|^2}, \quad (27)$$

where $\gamma = \alpha^2 + \beta_0^2$.
Power spectral density function of output we can obtain from the relation:

$$S_y(\omega) = S_p(\omega)|A(i\omega)|^2, \quad (28)$$

where for our case the square of the response function is subsequent:

$$|A(i\omega)|^2 = \frac{1}{|(i\omega)^2+a_1(i\omega)+a_0|^2} . \quad (29)$$

So we obtain:

$$S_y(\omega) = \frac{D\alpha}{\pi}\frac{\omega^2+\gamma}{|(i\omega)^2+2\alpha(i\omega)+\gamma|^2|(i\omega)^2+a_1(i\omega)+a_0|^2} \quad (30)$$

Dispersion of the output process we can obtain from integral:

$$D_y = \int_{-\infty}^{\infty} S_y(\omega)d\omega . \quad (31)$$

By substitution of the (30) in the (31) and integration we obtain:

$$D_y = \frac{D\alpha(a_0g_1+g_1g_2-g_3)}{a_0(g_1g_2g_3-a_0g_1^2\gamma-g_3^2)}, \quad (32)$$

where $g_1 = a_1 + 2\alpha$, $g_2 = a_0 + 2\alpha a_1 + \gamma$ and $g_3 = 2\alpha a_0 + \gamma a_1$.

2.3 Excitation No. 3

Finally, let us consider random excitation described by the correlation function of such type:

$$K_{pp}(\tau) = D(\cos\beta_0\tau + \frac{\alpha}{\beta_0}\sin\beta_0 \mid \tau \mid)e^{-\alpha|\tau|}. \tag{33}$$

The corresponding spectral density function will be:

$$S_p(\omega) = \frac{2D\alpha}{\pi}\frac{\gamma}{\mid (i\omega)^2 + 2\alpha(i\omega) + \gamma \mid^2}. \tag{34}$$

By implementing of the analogue operations as in the section 2.2 we obtain the subsequent relation for the dispersion of the output process:

$$D_y = \frac{2D\alpha(g_1g_2 - g_3)}{a_0(g_1g_2g_3 - a_0g_1^2\gamma - g_3^2)}, \tag{35}$$

Further for obtaining of the standard values of solution the relations (24), (32) and (35) are used for accuracy evaluation of the software tools.

3 Numerical Results

Let us set the subsequent values of excitation parameters: D = 60000, α = 1, β_0 = 3. In accordance with Ansys user methodic [7] random excitation we have to describe by the subsequent table of power spectral density. Maximal number of the value pairs is 50 that were used for higher accuracy. Approximations of the power spectral density functions of excitations in frequency region 0 to 10 Hz are shown on the figure 2.
The parameters of equation of motion (5) are supposed as following: a_0 = 900, but dissipation a_1 are varied in region 0.1 to 59.999.
Ratio of standard deviations of the output process y obtained by Ansys and Imita to the exact solutions obtained by relations (24) (32) and (35) are shown on the figure 3.

4 Conclusions

A comparison of accuracy of the proposed (Imita) and the conventional power spectral density method (Ansys) is performed. It is shown that for the examined cases of narrow band random excitation, precision of standard deviation of the generalised coordinate obtained by

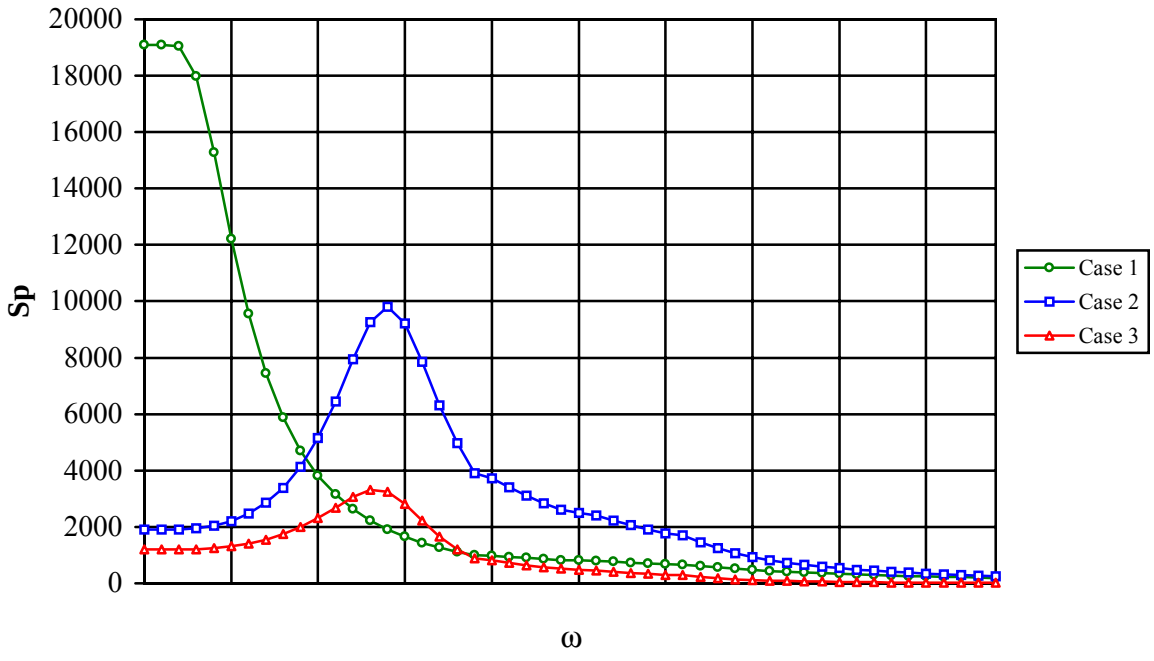

Figure 2: Spectral density functions of excitations 1, 2 and 3

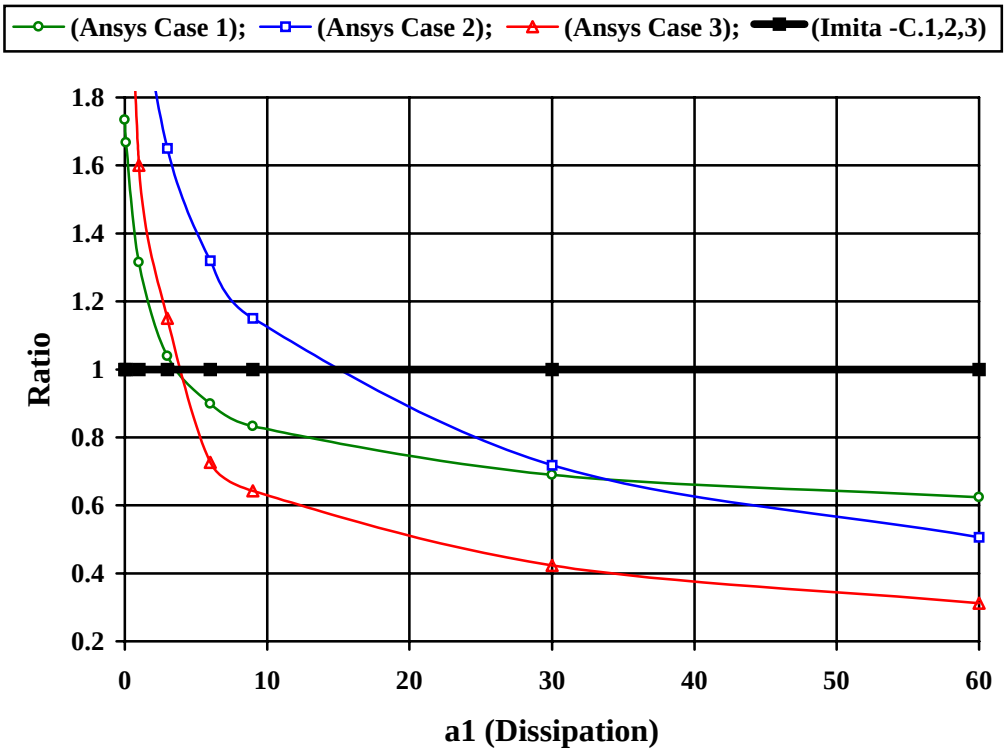

Figure 3: Comparison of accuracy of obtained results

Ansys essentially depends from dissipation in the dynamic system. During variation of the value of dissipation from lightly to critical damping of the system, errors of the results obtained by Ansys substantially increase. For the values of dissipation equal or greater than critical damping Ansys does not allow obtaining of any results. Also it is assumed that investigated dynamic systems have only classical distribution of damping.
Imita has not such restrictions as well as accuracy of obtained results does not depend from the value of dissipation. Also from the computational point of view proposed approach is very effective because for obtaining of the results it needs to use well-developed linear algebra methods instead of time-consuming integration procedures.
Critical loss of precision of Ansys appears for very small values of dissipation. So using of Ansys processor for optimisation in such type tasks is very problematic.
At the same time we have to note that before recommendation of the random vibration solution processor of Imita in the automatic procedure of optimisation we need to investigate behaviour of this processor in the tests of random vibration for multibody systems. These questions are under investigation now.

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